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Research Article

Course Control of Underactuated Ship Based on Nonlinear Robust Neural Network Backstepping Method

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The problem of course control for underactuated surface ship is addressed in this paper. Firstly, neural networks are adopted to determine the parameters of the unknown part of ideal virtual backstepping control, even the weight values of neural network are updated by adaptive technique. Then uniform stability for the convergence of course tracking errors has been proven through Lyapunov stability theory. Finally, simulation experiments are carried out to illustrate the effectiveness of proposed control method.

1. Introduction

Tracking control performance for surface vessel along the predefined route has been an essential control problem for marine autopilot system design, and it has received considerable attractions from control community. In 1922, proportional-integral-derivative (PID) autopilot for ship steering was presented by Nicholas Minosky [1]. PID controller greatly improved the performance of autopilots. Until the 1980s almost all makes of autopilots were based on these controllers. One challenge for tracking control of surface vessel based on above method is that the systems are often underactuated by the sway motion due to weight, complexity, and efficiency considerations and exhibit nonholonomic constraints, which meets Brocket's theorem that there is no continuous or even smooth time-invariant state feedback law that can stabilize the system to the origin [2]. Another challenge is that the vessel model itself exhibits severe nonlinear characteristic and model uncertainties induced by the ocean environment [3, 4].

For the ship with nonlinear maneuvering characteristics and without uncertainties, a state feedback linearization control law was designed [5], while feedback linearization with saturation and slew rate limiting actuators was discussed [6]. Later, combined with a genetic algorithm, the backstepping method was employed to develop a nonlinear ship course controller by Witkowska and Smierzchalski [7], where the ship course parameters were automatically tuned to the optimal values with the aid of a genetic algorithm. Even considering the ship steering model with both constant parametric uncertainties and input disturbance with unknown bound, a robust adaptive nonlinear control law was presented based on projection approach and Lyapunov stability theory [8]. Recently many papers have tackled these problems based on Lyapunov theory [9–12]. In [13–15] a global tracking controller for underactuated ship is addressed with nonzero off-diagonal terms, the reference trajectory is generated by using a virtual target guidance algorithm, and the controller designed is facilitated by an introduction of changing the ship outputs, several coordinate transformations, and backstepping method. And the controller design is heavily depending on accurate dynamic model; the robustness against disturbance has not been addressed. A method using backstepping adaptive dynamical sliding mode control is presented for path following control of USV in [16], the control system takes account of the modeling errors and disturbances, and simplified tracking error dynamics are obtained by assuming that the sway velocity is small which can be neglected in the controller design and only for straight line path tracking can be achieved. The LOS based guidance law is also used in the controller design which causes the complexity of computing high-order derivative of virtual control. In [17], a transformation of vessel kinematics to the Serret-Frenet frame is introduced by exploring an extra degree of freedom by controlling explicitly the progression rate of the virtual target along the path and overcomes the major singular problem; approach angle is introduced for controller design via backstepping method. Neural networks are introduced to enhance system stability and transient performance, which can handle the known dynamics and uncertainties of systems well [18–20]. Particularly in [12] a single hidden layer neural network (SHLNN) is adopted to obtain the adaptive signal online, but the choice of the single hidden layer neural network is limited by the number of hidden layer node selections that will affect the online learning speed and accuracy and cannot produce a better estimation effect on the fast changing disturbances.

Therefore, a solution to the course control of underactuated surface vessel is addressed in this paper. In view of the characteristics of the underactuated performance, the backstepping control method is used to deal with above problem. The direct adaptive neural network is adopted to design control law by using the RBF neural network to overcome the problem that the ideal virtual control cannot be used directly in practice. The weights of the neural network are updated by adaptive technique to guarantee the stability of the closed-loop system through Lyapunov stability theory. Simulation results are illustrated to verify the performance of the proposed adaptive neural network controller with good precision.

2. Adaptive Robust Neural Network Controller Design

2.1. Problem Description. Consider the following nonlinear systems:

$$\begin{aligned} \dot{x}_i &= f_i(\overline{x}_i) + g_i(\overline{x}_i)x_{i+1} + d_i, \quad 1 \le i \le n-1, \\ \dot{x}_n &= f_n(\overline{x}_n) + g_n(\dot{x}_n)u + d_n, \quad n \ge 2, \\ y &= x_1, \end{aligned} \tag{1}$$

where $\overline{x}_i = [x_1, x_2, \dots, x_i]$ is system state, u is control input, and y is system output. The control objective is to design an adaptive neural network controller and make y track y_d. y_d meets the smooth bounded reference model as follows:

$$\begin{aligned} \dot{x}_{di} &= f_{di}(x_d), \quad 1 \le i \le m \\ y_d &= x_{d1}, \quad m \ge n, \end{aligned} \tag{2}$$

where $x_d = [x_{d1}, x_{d2}, \dots, x_{dm}]^T \in R^m$ is state constant, $y_d \in R$ represents system output, and $f_{di}(\cdot)$, $i = 1, 2, \dots, m$, denote nonlinear function, assuming that the reference model for each state is bounded as $x_d \in \Omega_d$, $\forall t \ge 0$.

Assumption 1. There is an unknown constant p_i^* to meet, $\forall(\overline{x}_n, t) \in R^n \times R^+$, $|d_i(\overline{x}_n, t)| \le p_i^* \rho_i(\overline{x}_i)$, and $\rho_i(\overline{x}_i)$ is a known positive smooth function.

2.2. Direct Adaptive Neural Network Controller Design. In view of the problems and solutions described in the last section, the direct adaptive neural network controller for nonlinear systems with RBF neural network is chosen. Detailed design steps will be described in the following.

Step 1. Let $z_1 = x_1 - x_{d1}$, $z_2 = x_2 - \alpha_1$, and then

$$\dot{z}_1 = f_1(x_1) + g_1(x_1)x_2 + d_1 - \dot{x}_{d1}. \tag{3}$$

Consider the following Lyapunov function:

$$V_1 = \frac{1}{2g_1(x_1)}z_1^2 + \frac{1}{2}\widetilde{W}_1^T\Gamma_1^{-1}\widetilde{W}_1, \tag{4}$$

where $\widetilde{W}_1 = \widehat{W}_1 - W_1^*$, W_1^* represents the ideal weight vector of neural network, $\widehat{W}_1$ represents the estimated value of the neural network weight vector, $\widetilde{W}_1$ represents the estimation error of weight vector, $\Gamma_1 = \Gamma_1^T > 0$ is the adaptive gain matrix, and the derivation of V_1 can be computed as

$$\begin{aligned} \dot{V}_1 &= \frac{z_1\dot{z}_1}{g_1(x_1)} + \frac{\dot{g}_1(x_1)z_1^2}{2g_1^2(x_1)} + \widetilde{W}_1^T\Gamma_1^{-1}\dot{\widehat{W}}_1 \\ &= \frac{z_1}{g_1(x_1)}\left(f_1(x_1) + g_1(x_1)x_2 + d_1 - \dot{x}_{d1}\right) \\ &\quad + \frac{\dot{g}_1(x_1)z_1^2}{2g_1^2(x_1)} + \widetilde{W}_1^T\Gamma_1^{-1}\dot{\widehat{W}}_1 \\ &= z_1\left(z_2 + \alpha_1 + \frac{f_1(x_1) - \dot{x}_{d1}}{g_1(x_1)}\right) + \frac{z_1 d_1}{g_1(x_1)} \\ &\quad + \frac{\dot{g}_1(x_1)z_1^2}{2g_1^2(x_1)} + \widetilde{W}_1^T\Gamma_1^{-1}\dot{\widehat{W}}_1. \end{aligned} \tag{5}$$

According to Assumption 1, we can get

$$\begin{aligned} \dot{V}_1 &\le z_1\left(z_2 + \alpha_1 + \frac{f_1(x_1) - \dot{x}_{d1}}{g_1(x_1)}\right) + \frac{z_1^2\rho_1^2}{2g_1^2(x_1)} + \frac{P_1^{*2}}{2} \\ &\quad + \frac{\dot{g}_1(x_1)z_1^2}{2g_1^2(x_1)} + \widetilde{W}_1^T\Gamma_1^{-1}\dot{\widehat{W}}_1 \\ &= z_1\left(z_2 + \alpha_1 + \frac{f_1(x_1) - \dot{x}_{d1}}{g_1(x_1)} + \frac{z_1\rho_1^2}{2g_1^2(x_1)}\right) + \frac{P_1^{*2}}{2} \\ &\quad + \frac{\dot{g}_1(x_1)z_1^2}{2g_1^2(x_1)} + \widetilde{W}_1^T\Gamma_1^{-1}\dot{\widehat{W}}_1. \end{aligned} \tag{6}$$

There is an ideal virtual feedback control law:

$$\alpha_1^* = -c_1 z_1 - \left[\frac{f_1(x_1) - \dot{x}_{d1}}{g_1(x_1)} + \frac{z_1\rho_1^2}{2g_1^2(x_1)}\right], \tag{7}$$

where $c_1 > 0$ is designed controller parameter.

Because of the unknown smooth functions $f_1(x_1)$ and $g_1(x_1)$, we cannot actually get the ideal feedback control law α_1^*; from (7) we can see that the unknown part $(f_1(x_1) - \dot{x}_{d1})/g_1(x_1)$ is smooth function of x_1 and $\dot{x}_{d1}$, so that

$$\begin{aligned} h_1(Z_1) &\triangleq \frac{f_1(x_1) - \dot{x}_{d1}}{g_1(x_1)} + \frac{z_1\rho_1^2}{2g_1^2(x_1)}, \\ Z_1 &\triangleq [x_1, \dot{x}_{d1}]^T \subset R^2. \end{aligned} \tag{8}$$

RBF neural network $W_1^T S_1(Z_1)$ is used to approximate the unknown function $h_1(Z_1)$, and α_1^* can be expressed as

$$\alpha_1^* = -c_1 z_1 - W_1^{*T} S_1(Z_1) - e_1, \tag{9}$$

where $|e_1| \le e_1^*$ is estimated error and meets $e_1^* > 0$.

Because W_1^* is unknown, the virtual control law is selected as follows:

$$\alpha_1 = -c_1 z_1 - \widehat{W}_1^T S_1(Z_1) \tag{10}$$

and then

$$\begin{aligned}\dot{V}_1 \le{} & z_1 z_2 - c_1 z_1^2 + \frac{\dot{g}_1(x_1) z_1^2}{2g_1^2(x_1)} + z_1 e_1 + \frac{P_1^{*2}}{2} \\ & - \widetilde{W}_1^T S_1 z_1 + \widetilde{W}_1^T \Gamma_1^{-1} \dot{\widehat{W}}_1.\end{aligned} \tag{11}$$

Adaptive law can be chosen as follows:

$$\dot{\widehat{W}}_1 = \dot{\widetilde{W}}_1 = \Gamma_1 \left[S_1(Z_1) z_1 - \sigma_1 \widehat{W}_1\right], \tag{12}$$

where $\sigma_1 > 0$ and then

$$\begin{aligned}\dot{V}_1 \le{} & z_1 z_2 - c_1 z_1^2 + \frac{\dot{g}_1(x_1) z_1^2}{2g_1^2(x_1)} + z_1 e_1 + \frac{P_1^{*2}}{2} \\ & - \sigma_1 \widetilde{W}_1^T \widehat{W}_1.\end{aligned} \tag{13}$$

Let $c_1 = c_{10} + c_{11}$, where $c_{10} > 0$ and $c_{11} > 0$, and then the upper equation becomes

$$\begin{aligned}\dot{V}_1 \le{} & z_1 z_2 - \left(c_{10} + \frac{\dot{g}_1}{2g_1^2}\right) z_1^2 - c_{11} z_1^2 + z_1 e_1 + \frac{P_1^{*2}}{2} \\ & - \sigma_1 \widetilde{W}_1^T \widehat{W}_1.\end{aligned} \tag{14}$$

According to the complete square formula,

$$\begin{aligned}-\sigma_1 \widetilde{W}_1^T \widehat{W}_1 &= -\sigma_1 \widetilde{W}_1^T \left(\widetilde{W}_1 + W_1^*\right) \\ &\le -\sigma_1 \left\|\widetilde{W}_1\right\|^2 + \sigma_1 \left\|\widetilde{W}_1\right\| \left\|W_1^*\right\| \\ &\le -\frac{\sigma_1 \left\|\widetilde{W}_1\right\|^2}{2} + \frac{\sigma_1 \left\|W_1^*\right\|^2}{2},\end{aligned} \tag{15}$$

$$-c_{11} z_1^2 + z_1 e_1 \le -c_{11} z_1^2 + z_1 \left|e_1\right| \le \frac{e_1^2}{4c_{11}} \le \frac{e_1^{*2}}{4c_{11}}.$$

Because $-(c_{10} + (\dot{g}_1/2g_1^2)) z_1^2 \le -(c_{10} - (g_{1d}/2g_{1m}^2)) z_1^2$, we can make $(c_{10}^* \triangleq c_{10} - (g_{1d}/2g_{1m}^2)) > 0$ by choosing the appropriate c_{10} and obtain the following inequality:

$$\begin{aligned}\dot{V}_1 \le{} & z_1 z_2 - c_{10}^* z_1^2 - \frac{\sigma_1 \left\|\widetilde{W}_1\right\|^2}{2} + \frac{\sigma_1 \left\|W_1^*\right\|^2}{2} + \frac{e_1^{*2}}{4c_{11}} \\ & + \frac{P_1^{*2}}{2}.\end{aligned} \tag{16}$$

The cross coupling $z_1 z_2$ in (16) will be eliminated in the next step.

Step 2. Let $z_2 = x_2 - \alpha_1$; then

$$\dot{z}_2 = f_2(\bar{x}_2) + g_2(\bar{x}_2) x_3 + d_2 - \dot{\alpha}_1. \tag{17}$$

From (10) we can see that α_1 is a function of x_1, x_d, and $\widehat{W}_1$, and $\dot{\alpha}_1$ can be written as

$$\begin{aligned}\dot{\alpha}_1 &= \frac{\partial \alpha_1}{\partial x_1} \dot{x}_1 + \frac{\partial \alpha_1}{\partial x_d} \dot{x}_d + \frac{\partial \alpha_1}{\partial \widehat{W}_1} \dot{\widehat{W}}_1 \\ &= \frac{\partial \alpha_1}{\partial x_1} \left(g_1(x_1) x_2 + f_1(x_1)\right) + \varphi_1,\end{aligned} \tag{18}$$

where $\phi_1 = (\partial \alpha_1/\partial x_d) \dot{x}_d + (\partial \alpha_1/\partial \widehat{W}_1)[\Gamma_1(S_1(Z_1) z_1 - \sigma_1 \widehat{W}_1)]$ can be calculated.

Consider the following Lyapunov function:

$$V_2 = V_1 + \frac{1}{2g_2(\bar{x}_2)} z_2^2 + \frac{1}{2} \widetilde{W}_2^T \Gamma_2^{-1} \widetilde{W}_2, \tag{19}$$

where $\Gamma_2 = \Gamma_2^T > 0$ is an adaptive gain matrix.

Then the derivation of V_2 can be calculated as

$$\begin{aligned}\dot{V}_2 &= \dot{V}_1 + \frac{z_2 \dot{z}_2}{g_2(\bar{x}_2)} + \frac{\dot{g}_2(\bar{x}_2) z_2^2}{2g_2^2(\bar{x}_2)} + \widetilde{W}_2^T \Gamma_2^{-1} \dot{\widehat{W}}_2 \\ &= \dot{V}_1 + \frac{z_2}{g_2(\bar{x}_2)} \left(f_2(\bar{x}_2) + g_2(\bar{x}_2) x_3 + d_2 - \dot{\alpha}_1\right) \\ &\quad + \frac{\dot{g}_2(\bar{x}_2) z_2^2}{2g_2^2(\bar{x}_2)} + \widetilde{W}_2^T \Gamma_2^{-1} \dot{\widehat{W}}_2 \\ &= \dot{V}_1 + z_2 \left(z_3 + \alpha_2 + \frac{f_2(\bar{x}_2) - \dot{\alpha}_1}{g_2(\bar{x}_2)}\right) + \frac{z_2 d_2}{g_2(\bar{x}_2)} \\ &\quad + \frac{\dot{g}_2(\bar{x}_2) z_2^2}{2g_2^2(\bar{x}_2)} + \widetilde{W}_2^T \Gamma_2^{-1} \dot{\widehat{W}}_2.\end{aligned} \tag{20}$$

According to Assumption 1 we can get

$$\begin{aligned}\dot{V}_2 &\le \dot{V}_1 + z_2 \left(z_3 + \alpha_2 + \frac{f_2(\bar{x}_2) - \dot{\alpha}_1}{g_2(\bar{x}_2)}\right) + \frac{z_2^2 \rho_2^2}{2g_2^2(\bar{x}_2)} \\ &\quad + \frac{P_2^{*2}}{2} + \frac{\dot{g}_2(\bar{x}_2) z_2^2}{2g_2^2(\bar{x}_2)} + \widetilde{W}_2^T \Gamma_2^{-1} \dot{\widehat{W}}_2 \\ &= \dot{V}_1 + z_2 \left(z_3 + \alpha_2 + \frac{f_2(\bar{x}_2) - \dot{\alpha}_1}{g_2(\bar{x}_2)} + \frac{z_2 \rho_2^2}{2g_2^2(\bar{x}_2)}\right) \\ &\quad + \frac{P_2^{*2}}{2} + \frac{\dot{g}_2(\bar{x}_2) z_2^2}{2g_2^2(\bar{x}_2)} + \widetilde{W}_2^T \Gamma_2^{-1} \dot{\widehat{W}}_2.\end{aligned} \tag{21}$$

There is an ideal feedback control law:

$$\alpha_2^* = -z_1 - c_2 z_2 - \left[\frac{f_2(\overline{x}_2) - \dot{\alpha}_1}{g_2(\overline{x}_2)} + \frac{z_2 \rho_2^2}{2g_2^2(\overline{x}_2)}\right], \tag{22}$$

where $c_2 > 0$ is a designed controller parameter.

Because of the unknown smooth functions $f_2(\overline{x}_2)$ and $g_2(\overline{x}_2)$, we cannot actually get the ideal feedback control law α_2^*; from (22) we can see that the unknown part is a smooth function of $\overline{x}_2$ and $\dot{\alpha}_1$; let

$$h_2(Z_2) \triangleq \frac{f_2(\overline{x}_2) - \dot{\alpha}_1}{g_2(\overline{x}_2)} + \frac{z_2 \rho_2^2}{2g_2^2(\overline{x}_2)}, \tag{23}$$

where $Z_2 \triangleq [\overline{x}_2^T, (\partial\alpha_1/\partial x_1), \phi_1]^T \subset R^4$. RBF neural network $W_2^T S_2(Z_2)$ is used to approximate the unknown function $h_2(Z_2)$, and α_2^* can be expressed as

$$\alpha_2^* = -z_1 - c_2 z_2 - W_2^{*T} S_2(Z_2) - e_2, \tag{24}$$

where W_2^* is expressed as the ideal constant weight vector and $|e_2| \le e_2^*$ is the estimated error and meets $e_2^* > 0$.

Because W_2^* is unknown, select the following virtual control law:

$$\alpha_2 = -z_1 - c_2 z_2 - \widehat{W}_2^T S_2(Z_2), \tag{25}$$

where $\widehat{W}_2$ is the estimated value of W_2^*; then

$$\begin{aligned}\dot{V}_2 \le{}& \dot{V}_1 - z_1 z_2 + z_2 z_3 - c_2 z_2^2 + \frac{\dot{g}_2(\overline{x}_2) z_2^2}{2g_2^2(\overline{x}_2)} + z_2 e_2 \\ &+ \frac{P_2^{*2}}{2} - \widetilde{W}_2^T S_2 z_2 + \widetilde{W}_2^T \Gamma_2^{-1} \dot{\widehat{W}}_2,\end{aligned} \tag{26}$$

where $\widetilde{W}_2 = \widehat{W}_2 - W_2^*$.

Adaptive law can be chosen as

$$\dot{\widehat{W}}_2 = \dot{\widetilde{W}}_2 = \Gamma_2 \left[S_2(Z_2) z_2 - \sigma_2 \widehat{W}_2\right], \tag{27}$$

where $\sigma_2 > 0$; then

$$\begin{aligned}\dot{V}_2 \le{}& \dot{V}_1 - z_1 z_2 + z_2 z_3 - c_2 z_2^2 + \frac{\dot{g}_2(\overline{x}_2) z_2^2}{2g_2^2(\overline{x}_2)} + z_2 e_2 \\ &+ \frac{P_2^{*2}}{2} - \sigma_2 \widetilde{W}_2^T \widehat{W}_2.\end{aligned} \tag{28}$$

Let $c_2 = c_{20} + c_{21}$, $c_{20}, c_{21} > 0$; then the upper equation becomes

$$\begin{aligned}\dot{V}_2 \le{}& \dot{V}_1 - z_1 z_2 + z_2 z_3 - \left(c_{20} + \frac{\dot{g}_2(\overline{x}_2)}{2g_2^2(\overline{x}_2)}\right) z_2^2 - c_{21} z_2^2 \\ &+ z_2 e_2 + \frac{P_2^{*2}}{2} - \sigma_2 \widetilde{W}_2^T \widehat{W}_2.\end{aligned} \tag{29}$$

According to the complete square formula,

$$\begin{aligned}-\sigma_2 \widetilde{W}_2^T \widehat{W}_2 &= -\sigma_2 \widetilde{W}_2^T \left(\widetilde{W}_2 + W_2^*\right) \\ &\le -\sigma_2 \left\|\widetilde{W}_2\right\|^2 + \sigma_2 \left\|\widetilde{W}_2\right\| \left\|W_2^*\right\| \\ &\le -\frac{\sigma_2 \left\|\widetilde{W}_2\right\|^2}{2} + \frac{\sigma_2 \left\|W_2^*\right\|^2}{2}, \\ -c_{21} z_2^2 + z_2 e_2 &\le -c_{21} z_2^2 + z_2 \left|e_2\right| \le \frac{e_2^2}{4c_{21}} \le \frac{e_2^{*2}}{4c_{21}}.\end{aligned} \tag{30}$$

Because $-(c_{20} + (\dot{g}_2/2g_2^2))z_2^2 \le -(c_{20} - (g_{2d}/2g_{2m}^2))z_2^2$, then we can make $(c_{20}^* \triangleq c_{20} - (g_{2d}/2g_{2m}^2)) > 0$ by selecting the proper c_{20}; then

$$\begin{aligned}\dot{V}_2 \le{}& \dot{V}_1 - z_1 z_2 + z_2 z_3 - c_{20}^* z_2^2 - \frac{\sigma_2 \left\|\widetilde{W}_2\right\|^2}{2} \\ &+ \frac{\sigma_2 \left\|W_2^*\right\|^2}{2} + \frac{e_2^{*2}}{4c_{21}} + \frac{P_2^{*2}}{2} \\ \le{}& z_2 z_3 - \sum_{k=1}^{2} c_{k0}^* z_k^2 - \sum_{k=1}^{2} \frac{\sigma_k \left\|\widetilde{W}_k\right\|^2}{2} + \sum_{k=1}^{2} \frac{\sigma_k \left\|W_k^*\right\|^2}{2} \\ &+ \sum_{k=1}^{2} \frac{e_k^{*2}}{4c_{k1}}.\end{aligned} \tag{31}$$

The cross coupling $z_2 z_3$ in (31) will be eliminated in the next step.

Step i ($3 \le i \le n-1$). The derivative of $z_i = x_i - \alpha_{i-1}$ can be calculated as

$$\dot{z}_i = f_i(\overline{x}_i) + g_i(\overline{x}_i) x_{i+1} - \dot{\alpha}_{i-1}, \tag{32}$$

where

$$\begin{aligned}\dot{\alpha}_{i-1} &= \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k} \left(g_k(\overline{x}_k) x_{k+1} + f_k(\overline{x}_k)\right) + \varphi_{i-1}, \\ \phi_{i-1} &= \sum_{k=1}^{i-1} \left(\frac{\partial \alpha_{i-1}}{\partial x_d}\right) \dot{x}_d \\ &\quad + \sum_{k=1}^{i-1} \left(\frac{\partial \alpha_{i-1}}{\partial \widehat{W}_k}\right) \left[\Gamma_k \left(S_k(Z_k) z_k - \sigma_k \widehat{W}_k\right)\right].\end{aligned} \tag{33}$$

Consider the following Lyapunov function:

$$V_i = V_{i-1} + \frac{1}{2g_i(\overline{x}_i)} z_i^2 + \frac{1}{2} \widetilde{W}_i^T \Gamma_i^{-1} \widetilde{W}_i, \tag{34}$$

where $\Gamma_i = \Gamma_i^T > 0$ is an adaptive gain matrix.

Then the derivation of V_i can be calculated as

$$\begin{aligned}\dot{V}_i &= \dot{V}_{i-1} + \frac{z_i\dot{z}_i}{g_i(\overline{x}_i)} + \frac{\dot{g}_i(\overline{x}_i) z_i^2}{2g_i^2(\overline{x}_i)} + \widetilde{W}_i^T\Gamma_i^{-1}\dot{\widehat{W}}_i \\ &= \dot{V}_{i-1} + \frac{z_i}{g_i(\overline{x}_i)}\left(f_i(\overline{x}_i) + g_i(\overline{x}_i)x_{i+1} + d_i - \dot{\alpha}_{i-1}\right) \\ &\quad + \frac{\dot{g}_i(\overline{x}_i) z_i^2}{2g_i^2(\overline{x}_i)} + \widetilde{W}_i^T\Gamma_i^{-1}\dot{\widehat{W}}_i \\ &= \dot{V}_{i-1} + z_i\left(z_{i+1} + \alpha_i + \frac{f_i(\overline{x}_i) - \dot{\alpha}_{i-1}}{g_i(\overline{x}_i)}\right) + \frac{z_i d_i}{g_i(\overline{x}_i)} \\ &\quad + \frac{\dot{g}_i(\overline{x}_i) z_i^2}{2g_i^2(\overline{x}_i)} + \widetilde{W}_i^T\Gamma_i^{-1}\dot{\widehat{W}}_i.\end{aligned} \tag{35}$$

According to Assumption 1 we can get

$$\begin{aligned}\dot{V}_i &\leq \dot{V}_{i-1} + z_i\left(z_{i+1} + \alpha_i + \frac{f_i(\overline{x}_i) - \dot{\alpha}_{i-1}}{g_i(\overline{x}_i)}\right) \\ &\quad + \frac{z_i^2\rho_i^2}{2g_i^2(\overline{x}_i)} + \frac{P_i^{*2}}{2} + \frac{\dot{g}_i(\overline{x}_i) z_i^2}{2g_i^2(\overline{x}_i)} + \widetilde{W}_i^T\Gamma_i^{-1}\dot{\widehat{W}}_i \\ &= \dot{V}_{i-1} \\ &\quad + z_i\left(z_{i+1} + \alpha_i + \frac{f_i(\overline{x}_i) - \dot{\alpha}_{i-1}}{g_i(\overline{x}_i)} + \frac{z_i^2\rho_i^2}{2g_i^2(\overline{x}_i)}\right) \\ &\quad + \frac{P_i^{*2}}{2} + \frac{\dot{g}_i(\overline{x}_i) z_i^2}{2g_i^2(\overline{x}_i)} + \widetilde{W}_i^T\Gamma_i^{-1}\dot{\widehat{W}}_i.\end{aligned} \tag{36}$$

There is an ideal feedback control law as

$$\alpha_i^* = -z_{i-1} - c_i z_i - \left[\frac{f_i(\overline{x}_i) - \dot{\alpha}_{i-1}}{g_i(\overline{x}_i)} + \frac{z_i\rho_i^2}{2g_i^2(\overline{x}_i)}\right], \tag{37}$$

where $c_i > 0$ is designed controller parameter.

Because of the unknown smooth functions $f_i(\overline{x}_i)$ and $g_i(\overline{x}_i)$, we cannot actually get the ideal feedback control law α_i^*; from (37) we can see that the unknown part is a smooth function of $\overline{x}_i$ and $\dot{\alpha}_{i-1}$, and let

$$h_i(Z_i) \triangleq \frac{f_i(\overline{x}_i) - \dot{\alpha}_{i-1}}{g_i(\overline{x}_i)} + \frac{z_i\rho_i^2}{2g_i^2(\overline{x}_i)}, \tag{38}$$

where

$$Z_i \triangleq \left[\overline{x}_i^T, \frac{\partial\alpha_{i-1}}{\partial x_1}, \dots, \frac{\partial\alpha_{i-1}}{\partial x_{i-1}}, \varphi_{i-1}\right]^T \subset R^{2i}. \tag{39}$$

By introducing the direct variable $(\partial\alpha_{i-1}/\partial x_1), \dots, (\partial\alpha_{i-1}/\partial x_{i-1})$, φ_{i-1}, we can make the number of neural networks minimized. RBF neural network $W_i^T S_i(Z_i)$ is used to approximate the unknown function $h_i(Z_i)$, and α_i^* can be expressed as

$$\alpha_i^* = -z_{i-1} - c_i z_i - W_i^{*T} S_i(Z_i) - e_i, \tag{40}$$

where $|e_i| \leq e_i^*$ is estimated error and meets $e_i^* > 0$.

Because W_i^* is unknown, select the following virtual control law:

$$\alpha_i = -z_{i-1} - c_i z_i - \widehat{W}_i^T S_i(Z_i), \tag{41}$$

where W_i^* is the estimated value of $\widehat{W}_i$; then

$$\begin{aligned}\dot{V}_i &\leq \dot{V}_{i-1} - z_{i-1}z_i + z_i z_{i+1} - c_i z_i^2 + \frac{\dot{g}_i(\overline{x}_i) z_i^2}{2g_i^2(\overline{x}_i)} + z_i e_i \\ &\quad + \frac{P_i^{*2}}{2} - \widetilde{W}_i^T S_i z_i + \widetilde{W}_i^T\Gamma_i^{-1}\dot{\widehat{W}}_i,\end{aligned} \tag{42}$$

where $\widetilde{W}_i = \widehat{W}_i - W_i^*$.

The following adaptive law can be selected as

$$\dot{\widehat{W}}_i = \dot{\widetilde{W}}_i = \Gamma_i\left[S_i(Z_i) z_i - \sigma_i\widehat{W}_i\right], \tag{43}$$

where $\sigma_i > 0$; then

$$\begin{aligned}\dot{V}_i &\leq \dot{V}_{i-1} - z_{i-1}z_i + z_i z_{i+1} - c_i z_i^2 + \frac{\dot{g}_i(\overline{x}_i) z_i^2}{2g_i^2(\overline{x}_i)} + z_i e_i \\ &\quad + \frac{P_i^{*2}}{2} - \sigma_i\widetilde{W}_i^T\widehat{W}_i.\end{aligned} \tag{44}$$

Let $c_i = c_{i0} + c_{i1}$, $c_{i0}, c_{i1} > 0$; then (44) can be rewritten as

$$\begin{aligned}\dot{V}_i &\leq \dot{V}_{i-1} - z_{i-1}z_i + z_i z_{i+1} - \left(c_{i0} + \frac{\dot{g}_i(\overline{x}_i)}{2g_i^2(\overline{x}_i)}\right) z_i^2 \\ &\quad - c_{i1}z_i^2 + z_i e_i + \frac{P_i^{*2}}{2} - \sigma_i\widetilde{W}_i^T\widehat{W}_i.\end{aligned} \tag{45}$$

According to the complete square formula,

$$\begin{aligned}-\sigma_i\widetilde{W}_i^T\widehat{W}_i &= -\sigma_i\widetilde{W}_i^T\left(\widetilde{W}_i + W_i^*\right) \\ &\leq -\sigma_i\left\|\widetilde{W}_i\right\|^2 + \sigma_i\left\|\widetilde{W}_i\right\|\left\|W_i^*\right\| \\ &\leq -\frac{\sigma_i\left\|\widetilde{W}_i\right\|^2}{2} + \frac{\sigma_i\left\|W_i^*\right\|^2}{2}, \\ -c_{i1}z_i^2 + z_i e_i &\leq -c_{i1}z_i^2 + z_i\left|e_i\right| \leq \frac{e_i^2}{4c_{i1}} \leq \frac{e_i^{*2}}{4c_{i1}}.\end{aligned} \tag{46}$$

Because $-(c_{i0} + (\dot{g}_i/2g_i^2))z_i^2 \le -(c_{i0} - (g_{id}/2g_{im}^2))z_i^2$, then we can make $(c_{i0}^* \triangleq c_{i0} - (g_{id}/2g_{im}^2)) > 0$ by selecting the proper c_{i0}; then

$$\begin{aligned}\dot{V}_i &\le \dot{V}_{i-1} - z_{i-1}z_i + z_i z_{i+1} - c_{i0}^* z_i^2 - \frac{\sigma_i \|\widetilde{W}_i\|^2}{2} \\ &\quad + \frac{\sigma_i \|W_i^*\|^2}{2} + \frac{e_i^{*2}}{4c_{i1}} + \frac{P_i^{*2}}{2} \\ &\le z_i z_{i+1} - \sum_{k=1}^{i} c_{k0}^* z_k^2 - \sum_{k=1}^{i} \frac{\sigma_k \|\widetilde{W}_k\|^2}{2} + \sum_{k=1}^{i} \frac{\sigma_k \|W_k^*\|^2}{2} \\ &\quad + \sum_{k=1}^{i} \frac{e_k^{*2}}{4c_{k1}} + \sum_{k=1}^{i} \frac{P_k^{*2}}{2}.\end{aligned} \tag{47}$$

The cross coupling $z_i z_{i+1}$ in (47) will be eliminated in the next step.

Step n. The derivative of $z_n = x_n - \alpha_{n-1}$ can be calculated as

$$\dot{z}_n = f_n(\bar{x}_n) + g_n(\bar{x}_{n-1})u - \dot{\alpha}_{n-1}, \tag{48}$$

where

$$\dot{\alpha}_{n-1} = \sum_{k=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_k}\left(g_k(\bar{x}_k)x_{k+1} + f_k(\bar{x}_k)\right) + \phi_{n-1}, \tag{49}$$

where

$$\begin{aligned}\phi_{n-1} &= \sum_{k=1}^{n-1}\left(\frac{\partial \alpha_{n-1}}{\partial x_d}\right)\dot{x}_d \\ &\quad + \sum_{k=1}^{n-1}\left(\frac{\partial \alpha_{n-1}}{\partial \widehat{W}_k}\right)\left[\Gamma_k\left(S_k(Z_k)z_k - \sigma_k \widehat{W}_k\right)\right].\end{aligned} \tag{50}$$

Consider the following Lyapunov function:

$$V_n = V_{n-1} + \frac{1}{2g_n(\bar{x}_n)}z_n^2 + \frac{1}{2}\widetilde{W}_n^T \Gamma_n^{-1}\widetilde{W}_n, \tag{51}$$

where $\Gamma_n = \Gamma_n^T > 0$ is an adaptive gain matrix. Then the derivation of V_n can be calculated as

$$\begin{aligned}\dot{V}_n &= \dot{V}_{n-1} + \frac{z_n \dot{z}_n}{g_i(\bar{x}_i)} + \frac{\dot{g}_n(\bar{x}_n)z_n^2}{2g_n^2(\bar{x}_n)} + \widetilde{W}_n^T \Gamma_n^{-1}\dot{\widehat{W}}_n \\ &= \dot{V}_{n-1} \\ &\quad + \frac{z_n}{g_n(\bar{x}_n)}\left(f_n(\bar{x}_n) + g_n(\bar{x}_n)u + d_n - \dot{\alpha}_{n-1}\right) \\ &\quad + \frac{\dot{g}_n(\bar{x}_n)z_n^2}{2g_n^2(\bar{x}_n)} + \widetilde{W}_n^T \Gamma_n^{-1}\dot{\widehat{W}}_n \\ &= \dot{V}_{n-1} + z_n\left(z_{n+1} + u + \frac{f_n(\bar{x}_n) - \dot{\alpha}_{n-1}}{g_n(\bar{x}_n)}\right) \\ &\quad + \frac{z_n d_n}{g_n(\bar{x}_n)} + \frac{\dot{g}_n(\bar{x}_n)z_n^2}{2g_n^2(\bar{x}_n)} + \widetilde{W}_n^T \Gamma_n^{-1}\dot{\widehat{W}}_n.\end{aligned} \tag{52}$$

According to Assumption 1 we can get

$$\begin{aligned}\dot{V}_n &\le \dot{V}_{n-1} + z_n\left(z_{n+1} + u + \frac{f_i(\bar{x}_i) - \dot{\alpha}_{n-1}}{g_i(\bar{x}_i)}\right) \\ &\quad + \frac{z_n^2\rho_n^2}{2g_n^2(\bar{x}_n)} + \frac{P_n^{*2}}{2} + \frac{\dot{g}_n(\bar{x}_n)z_n^2}{2g_n^2(\bar{x}_n)} + \widetilde{W}_n^T \Gamma_n^{-1}\dot{\widehat{W}}_n \\ &= \dot{V}_{n-1} \\ &\quad + z_n\left(z_{n+1} + u + \frac{f_n(\bar{x}_n) - \dot{\alpha}_{n-1}}{g_n(\bar{x}_n)} + \frac{z_n^2\rho_n^2}{2g_n^2(\bar{x}_n)}\right) \\ &\quad + \frac{P_n^{*2}}{2} + \frac{\dot{g}_n(\bar{x}_n)z_n^2}{2g_n^2(\bar{x}_n)} + \widetilde{W}_n^T \Gamma_n^{-1}\dot{\widehat{W}}_n.\end{aligned} \tag{53}$$

There is an ideal feedback control law as

$$u^* = -z_{i-1} - c_i z_i - \left[\frac{f_i(\bar{x}_i) - \dot{\alpha}_{i-1}}{g_i(\bar{x}_i)} + \frac{z_i\rho_i^2}{2g_i^2(\bar{x}_i)}\right], \tag{54}$$

where $c_n > 0$ is designed controller parameter.

Because of the unknown smooth functions $f_n(\bar{x}_n)$ and $g_i(\bar{x}_i)$, we cannot actually get the ideal feedback control law u^*; from (54) we can see the unknown part is a smooth function of $\bar{x}_n$ and $\dot{\alpha}_{n-1}$, and let

$$h_n(Z_i) \triangleq \frac{f_n(\bar{x}_n) - \dot{\alpha}_{n-1}}{g_n(\bar{x}_n)} + \frac{z_n\rho_n^2}{2g_n^2(\bar{x}_n)}, \tag{55}$$

where $Z_n \triangleq [\bar{x}_n^T, \partial\alpha_{n-1}/\partial x_1, \ldots, \partial\alpha_{n-1}/\partial x_{n-1}, \phi_{n-1}]^T \subset R^{2n}$.

RBF neural network $W_n^T S_n(Z_n)$ is used to approximate the unknown function $h_n(Z_n)$, and u^* can be expressed as

$$u^* = -z_{n-1} - c_n z_n - W_n^{*T} S_n(Z_n) - e_n, \tag{56}$$

where $|e_n| \le e_n^*$ is estimated error and meets $e_n^* > 0$.

Because W_n^* is unknown, select the following virtual control law:

$$u = -z_{n-1} - c_n z_n - \widehat{W}_n^T S_n(Z_n), \tag{57}$$

where $\widehat{W}_i$ is the estimated value of W_i^*; then

$$\begin{aligned}\dot{V}_n &\le \dot{V}_{n-1} - z_{n-1}z_n + z_n z_{n+1} - c_n z_n^2 + \frac{\dot{g}_n(\bar{x}_n)z_n^2}{2g_n^2(\bar{x}_n)} \\ &\quad + z_n e_n + \frac{P_n^{*2}}{2} - \widetilde{W}_n^T S_n z_n + \widetilde{W}_n^T \Gamma_n^{-1}\dot{\widehat{W}}_n,\end{aligned} \tag{58}$$

where $\widetilde{W}_n = \widehat{W}_n - W_n^*$.

The following adaptive law can be selected as

$$\dot{\widehat{W}}_n = \dot{\widetilde{W}}_n = \Gamma_n\left[S_n(Z_n)z_n - \sigma_n \widehat{W}_n\right], \tag{59}$$

where $\sigma_n > 0$; then

$$\begin{aligned}\dot{V}_n &\le \dot{V}_{n-1} - z_{n-1}z_n + z_n z_{n+1} - c_n z_n^2 + \frac{\dot{g}_n(\bar{x}_n)z_n^2}{2g_n^2(\bar{x}_n)} \\ &\quad + z_n e_n + \frac{P_n^{*2}}{2} - \sigma_n \widetilde{W}_n^T \widehat{W}_n.\end{aligned} \tag{60}$$

Let $c_n = c_{n0} + c_{n1}$, $c_{n0}, c_{n1} > 0$; (60) can be rewritten as

$$\begin{aligned}\dot{V}_n &\le \dot{V}_{n-1} - z_{n-1}z_n + z_n z_{n+1} - \left(c_{n0} + \frac{\dot{g}_n(\overline{x}_n)}{2g_n^2(\overline{x}_n)}\right) z_n^2 \\ &\quad - c_{n1}z_n^2 + z_n e_n + \frac{P_n^{*2}}{2} - \sigma_n \widetilde{W}_n^T \widehat{W}_n.\end{aligned} \tag{61}$$

According to the complete square formula,

$$\begin{aligned}-\sigma_n \widetilde{W}_n^T \widehat{W}_n &= -\sigma_n \widetilde{W}_n^T \left(\widetilde{W}_n + W_n^*\right) \\ &\le -\sigma_n \left\|\widetilde{W}_n\right\|^2 + \sigma_n \left\|\widetilde{W}_n\right\| \left\|W_n^*\right\| \\ &\le -\frac{\sigma_n \left\|\widetilde{W}_n\right\|^2}{2} + \frac{\sigma_n \left\|W_n^*\right\|^2}{2}, \\ -c_{n1}z_n^2 + z_n e_n &\le -c_{n1}z_n^2 + z_n \left|e_n\right| \le \frac{e_n^2}{4c_{n1}} \le \frac{e_n^{*2}}{4c_{n1}}.\end{aligned} \tag{62}$$

Because $-(c_{n0}+(\dot{g}_n/2g_n^2))z_n^2 \le -(c_{n0}-(g_{nd}/2g_{nm}^2))z_n^2$, then we can make $(c_{n0}^* \triangleq c_{n0} - (g_{nd}/2g_{nm}^2)) > 0$ by selecting the proper c_{n0}; then

$$\begin{aligned}\dot{V}_n &\le -\sum_{k=1}^n c_{k0}^* z_k^2 - \sum_{k=1}^n \frac{\sigma_k \left\|\widetilde{W}_k\right\|^2}{2} + \sum_{k=1}^n \frac{\sigma_k \left\|W_k^*\right\|^2}{2} \\ &\quad + \sum_{k=1}^n \frac{e_k^{*2}}{4c_{k1}} + \sum_{k=1}^n \frac{P_k^{*2}}{2}.\end{aligned} \tag{63}$$

Let $\delta \triangleq \sum_{k=1}^n(\sigma_k\|W_k^*\|^2/2) + \sum_{k=1}^n(e_k^{*2}/4c_{k1}) + \sum_{k=1}^n(p_k^{*2}/2)$, $c_{k0}^* \ge (\gamma/2g_{km})$, $c_{k0} > (\gamma/2g_{km}) + (g_{kd}/2g_{km}^2)$, $k = 1, 2, \ldots, n$, where $\gamma > 0$, $\sigma_k \ge \gamma\lambda_{\max}\{\Gamma_k^{-1}\}$, $k = 1, 2, \ldots, n$; then

$$\begin{aligned}\dot{V}_n &\le -\sum_{k=1}^n c_{k0}^* z_k^2 - \sum_{k=1}^n \frac{\sigma_k \left\|\widetilde{W}_k\right\|^2}{2} + \delta \\ &\le -\sum_{k=1}^n \frac{\gamma}{2g_{km}} z_k^2 - \sum_{k=1}^n \frac{\gamma \widetilde{W}_k^T \Gamma_k^{-1} \widetilde{W}_k}{2g_{km}} + \delta \\ &\le -\gamma \left[\sum_{k=1}^n \frac{1}{2g_k} z_k^2 + \sum_{k=1}^n \frac{\widetilde{W}_k^T \Gamma_k^{-1} \widetilde{W}_k}{2}\right] + \delta \\ &\le -\gamma V_n + \delta.\end{aligned} \tag{64}$$

The stability and control performance of the closed-loop adaptive system are demonstrated by the following theorem.

Theorem 2. *In the initial conditions, by formula (1), reference model (2), control law (57), and neural network weight update rate in (12), (27), (43), and (59), supposing that there is a large enough set of closed sets $\Omega_i \in R^{2i}$, $i = 1, 2, \ldots, n$, for any given moment $t \ge 0$, making $Z_i \in \Omega_i$, the following conclusions can be obtained as follows:*

(1) *The signal of the whole closed-loop system is bounded, and the state variable $\overline{x}_n$ and the neural network estimation errors $\widehat{W}_1^T, \ldots, \widehat{W}_n^T$ will eventually converge to the closed set as follows:*

$$\Omega_{s1} \triangleq \left\{\overline{x}_n, \widehat{W}_1, \ldots, \widehat{W}_n \mid V < \frac{\delta}{\gamma},\ x_d \in \Omega_d\right\}. \tag{65}$$

(2) *By choosing the proper control parameters, the output tracking error $y(t) - y_{d1}(t)$ is close to a small neighborhood of zero [21].*

3. Adaptive Robust Neural Network Control for Ship Course

3.1. Problem Formulation. This section introduces a simplified dynamic model of an underactuated surface vehicle with only one control input δ for heading control. A surface ship usually has three degrees of freedom for path following control in horizontal plane. Assuming that the vessel has three planes of symmetry, for most underactuated vessels have port/starboard symmetry, it can be neglected to simplify the vessel model for controller design. The detailed model which considers the environment disturbances can be set as follows:

$$\begin{aligned}\dot{y} &= U \sin\psi, \\ \dot{\psi} &= r, \\ \dot{r} &= -\frac{1}{T}r - \frac{\alpha}{T}r^3 + \frac{K}{T}\delta + \Delta, \\ y_1 &= y, \\ y_2 &= \psi,\end{aligned} \tag{66}$$

where y denotes transverse displacement in the earth inertial coordinates; $U = \sqrt{u^2 + v^2}$ is resultant velocity of ship; ψ is course angle; r is yawing angular velocity; K, T represent performance index for ship steering; α is coefficient of nonlinear term; δ is control rudder angle; y_1, y_2 represent system output.

The control objective is to design the controller δ to make the control output y, ψ achieve the setting value (y_d, ψ_d). Because the dimension of the system control input is less than the degree of freedom of the system, it is an underactuated system.

3.2. Dynamic Controller Design. Selection of coordinate transformation is as follows:

$$w_e = \psi + \arcsin\left(\frac{ky}{\sqrt{1 + (ky)^2}}\right). \tag{67}$$

The original system can be transformed into a single input single output system:

$$\begin{aligned}\dot{x}_1 &= \frac{k\dot{y}}{1 + (ky)^2} + x_2, \\ \dot{x}_2 &= -a_1 x_2 - a_2 {x_2}^3 + bu + \Delta,\end{aligned} \tag{68}$$

where $a_1 = 1/T$, $a_2 = \alpha/T$, $b = K/T$, $x_1 = w_e$, $x_2 = r$, $u = \delta$, and the output of whole system is x_1.

For system model (67) and (68), the controller design is carried out by using backstepping method.

Step 1. Let $z_1 = x_1$, $x_{d1} = 0$; then

$$\dot{z}_1 = \frac{k\dot{y}}{1 + (ky)^2} + x_2. \tag{69}$$

For the subsystem z_1, $\alpha_1^* \triangleq x_2$ is chosen as virtual control input. Select the Lyapunov function $V_{z1} = (1/2)z_1^2$, and there is

$$\dot{V}_{z1} = z_1\dot{z}_1 = \left(\frac{k\dot{y}}{1 + (ky)^2} + x_2\right)z_1. \tag{70}$$

Let $z_2 = x_2 - \alpha_1$; then $x_2 = z_2 + \alpha_1$,

$$\dot{V}_{z1} = \left(\frac{k\dot{y}}{1 + (ky)^2} + z_2 + \alpha_1\right)z_1. \tag{71}$$

Select the following virtual control law:

$$\alpha_1^* = -c_1 z_1 - \frac{k\dot{y}}{1 + (ky)^2}. \tag{72}$$

$\dot{V}_{z1} = z_1 z_2 - c_1 z_1^2$, because $k\dot{y}/(1 + (ky)^2)$ is unknown function, $h_1(Z_1) = k\dot{y}/(1 + (ky)^2)$, and we will adopt RBF NN to estimate $h_1(Z_1)$ and get $h_1(Z_1) = W_1^{*T}S_1(Z_1) + \varepsilon_1$. But the actual use of the NN for the system is $h_1(Z_1) = \widehat{W}_1^T S_1(Z_1)$. Actual virtual control input is $\alpha_1 = -c_1 z_1 - \widehat{W}_1^T S_1(Z_1)$; then

$$\begin{aligned}\dot{z}_1 &= \frac{k\dot{y}}{1 + (ky)^2} + z_2 + \alpha_1 \\ &= \left(z_2 - c_1 z_1 - \widetilde{W}_1 S_1(Z_1) + \varepsilon_1\right),\end{aligned} \tag{73}$$

where $\widetilde{W}_1 = \widehat{W}_1 - W_1^*$.

Select Lyapunov function as

$$V_1 = V_{z1} + \frac{1}{2}\widetilde{W}_1^T \Gamma^{-1}\widetilde{W}_1; \tag{74}$$

then

$$\begin{aligned}\dot{V}_1 &= \dot{V}_{z1} + \widetilde{W}_1\Gamma^{-1}\dot{\widehat{W}}_1 \le z_1\left(z_2 + \alpha_1 + h_1(Z_1)\right) \\ &= z_1\left[z_2 - c_1 z_1 - \widehat{W}_1 S_1(Z_1) + W_1^* S_1(Z_1) + \varepsilon_1\right] \\ &\quad + \widetilde{W}_1\Gamma^{-1}\dot{\widehat{W}}_1 \\ &= z_1\left[z_2 - c_1 z_1 - \widetilde{W}_1 S_1(Z_1) + \varepsilon_1\right] + \widetilde{W}_1\Gamma^{-1}\dot{\widehat{W}}_1.\end{aligned} \tag{75}$$

The adaptive law of neural network can be designed as

$$\dot{\widehat{W}}_1 = \dot{\widetilde{W}}_1 = \Gamma_1\left[S_1(Z_1)z_1 - \sigma_1\widehat{W}_1\right], \tag{76}$$

where $\sigma_1 > 0$. Let $c_1 = c_{10} + c_{11}$, where $c_{10}, c_{11} > 0$.

Furthermore,

$$\dot{V}_1 = z_1 z_2 - c_{10}z_1^2 - c_{11}z_1^2 + z_1\varepsilon_1 - \sigma_1\widetilde{W}_1^T\widehat{W}_1; \tag{77}$$

then

$$\begin{aligned}-\sigma_1\widetilde{W}_1^T\widehat{W}_1 &= -\sigma_1\widetilde{W}_1^T\left(\widetilde{W}_1 + W_1^*\right) \\ &\le -\sigma_1\left\|\widetilde{W}_1\right\|^2 + \sigma_1\left\|\widetilde{W}_1\right\|\left\|W_1^*\right\| \\ &\le -\frac{\sigma_1\left\|\widetilde{W}_1\right\|^2}{2} + \frac{\sigma_1\left\|W_1^*\right\|^2}{2}\end{aligned} \tag{78}$$

because

$$-c_{11}z_1^2 + z_1\varepsilon_1 \le -c_{11}z_1^2 + z_1\left|\varepsilon_1\right| \le \frac{\varepsilon_1^2}{4c_{11}} \le \frac{\varepsilon_1^{*2}}{4c_{11}}. \tag{79}$$

Finally we can get

$$\dot{V}_1 < z_1 z_2 - c_{10}^* z_1^2 - \frac{\sigma_1\left\|\widetilde{W}_1\right\|^2}{2} + \frac{\sigma_1\left\|W_1^*\right\|^2}{2} + \frac{\varepsilon_1^{*2}}{4c_{11}}. \tag{80}$$

Step 2. Let $z_2 = x_2 - \alpha_1$; derivation of z_2 can be calculated as

$$\begin{aligned}\dot{z}_2 &= f_2(\overline{x}_2) + g_2(\overline{x}_2)u + \Delta - \dot{\alpha}_1 \\ &= -a_1 x_2 - a_2 x_2{}^3 + bu + \Delta - \dot{\alpha}_1.\end{aligned} \tag{81}$$

Because $V_{z2} = (1/2b)z_2^2$, then

$$\begin{aligned}\dot{V}_{z2} &= \frac{1}{b}z_2\dot{z}_2 = \frac{1}{b}z_2\left(-a_1 x_2 - a_2 x_2{}^3 + bu + \Delta - \dot{\alpha}_1\right) \\ &= z_2\left[u + \frac{1}{b}\left(-a_1 x_2 - a_2 x_2{}^3 - \dot{\alpha}_1\right)\right] + \frac{\Delta}{b}z_2 \\ &\le z_2\left[u + \frac{1}{b}\left(-a_1 x_2 - a_2 x_2{}^3 - \dot{\alpha}_1 + \frac{\rho^2 z_2}{2b}\right)\right] \\ &\quad + \frac{p^2}{2},\end{aligned} \tag{82}$$

where $\Delta \le p \cdot \rho(x)$, p is unknown parameter, $\rho(x)$ is known nonlinear function, and then

$$u^* = -z_1 - c_2 z_2 - \frac{1}{b}\left(-a_1 x_2 - a_2 x_2{}^3 - \dot{\alpha}_1 + \frac{\rho^2 z_2}{2b}\right). \tag{83}$$

Let

$$h_2(Z_2) = \frac{1}{b}\left(-a_1 x_2 - a_2 x_2{}^3 - \dot{\alpha}_1 + \frac{\rho^2 z_2}{2b}\right). \tag{84}$$

Equation (83) can be rewritten as

$$u^* = -z_1 - c_2 z_2 - h_2(Z_2). \tag{85}$$

In the same way we use RBF NN estimate $h_2(Z_2)$:

$$h_2(Z_2) = W_2^{*T}S_2(Z_2) + \varepsilon_2. \tag{86}$$

The actual use of the NN for the system and controller can be expressed as

$$\begin{aligned} h_2(Z_2) &= \widehat{W}_2^T S_2(Z_2), \\ u &= z_1 - c_2 z_2 - \widehat{W}_2^T S_2(Z_2). \end{aligned} \quad (87)$$

Select Lyapunov function as

$$V_2 = V_1 + V_{z2} + \frac{1}{2}\widetilde{W}_2^T \Gamma^{-1} \widetilde{W}_2. \quad (88)$$

The derivation of V_2 can be calculated as

$$\begin{aligned} \dot{V}_2 &= \dot{V}_1 + \dot{V}_{z2} + \widetilde{W}_1 \Gamma^{-1} \dot{\widehat{W}}_1 \\ &\leq z_1 z_2 - c_{10}^* z_1^2 - \frac{\sigma_1 \|\widetilde{W}_1\|^2}{2} + \frac{\sigma_1 \|W_1^*\|^2}{2} + \frac{\varepsilon_1^{*2}}{4c_{11}} \\ &\quad + z_2 \left[-z_1 - c_2 z_2 - \widehat{W}_2 S_2(Z_2) + W_2^* S_2(Z_2) + \varepsilon_2 \right] \\ &\quad + \frac{p^2}{2} + \widetilde{W}_1 \Gamma^{-1} \dot{\widehat{W}}_1 \\ &= -\sum_{i=1}^{2} c_{i0}^* z_i^2 - \sum_{i=1}^{2} \frac{\sigma_i \|\widetilde{W}_i\|^2}{2} + \sum_{i=1}^{2} \frac{\sigma_i \|W_i^*\|^2}{2} + \sum_{i=1}^{2} \frac{\varepsilon_i^{*2}}{4c_{11}} \\ &\quad + \frac{p^2}{2}. \end{aligned} \quad (89)$$

Therefore, all signals in the close loop of course tracking system are stable, and the tracking errors can be made arbitrarily small by selecting appropriate controller parameters. So the final control law can be designed as

$$u = z_1 - c_2 z_2 - \widehat{W}_2^T S_2(Z_2). \quad (90)$$

4. Numerical Simulations and Analysis

The simulation experiment can be operated based on an experimental ship. The nonlinear mathematical model for the ship has been presented in [22], which captures the fundamental characteristics of dynamics and offers good maneuverability in the open-loop test. To illustrate the effectiveness of the theoretical results, the proposed control scheme is implemented and simulated with the above nonlinear model with tracking task.

The characteristic parameters of the ship used in the simulation are given as $K = 0.478$, $T = 216$, and $\alpha = 30$. Neural network contains 25 neurons; that is, $l_1 = 25$; the center vector μ_l $(l = 1, 2, \ldots, l_1)$ is uniformly distributed in the width $[-2, 2]\times[-2, 2]\times[-2, 2]$. Neural network $\widehat{W}_2^T S_2(Z_2)$ contains 135 neurons; that is, $l_2 = 125$; the center vector μ_l $(l = 1, 2, \ldots, l_2)$ is uniformly distributed in the width $[-4, 4] \times [-4, 4] \times [-4, 4] \times [-4, 4] \times [-4, 4] \times [-4, 4] \times [-4, 0] \times [-6, 6]$. The controller design parameters are given as follows which satisfy the condition mentioned in design procedure: $k = 0.1394$, $c_1 = 4$, $c_2 = 120$, $\Gamma_1 = \mathrm{diag}\{3\}$, $\Gamma_2 = \mathrm{diag}\{4\}$, and $\sigma_1 = 4$, $\sigma_2 = 2$. The initial linear and angular velocity of ship used in the simulation are given as $[u, v, r]^T = [0.1, 0, 0]^T$, $[x, y, \psi]^T = [10, 30, -\pi/4]^T$ is the initial position and orientation vector of ship, and the desired velocity of ship is given as $u_d = 1\,(\mathrm{m/s})$. We choose the reference trajectory as $10\cos\omega t$.

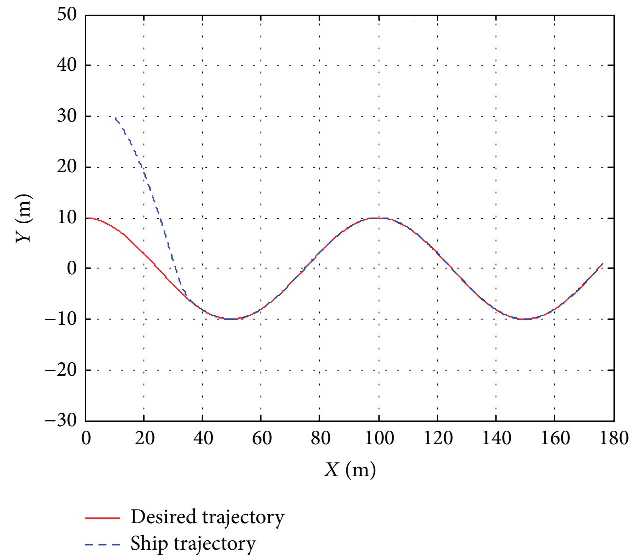

Figure 1: Ship tracking performance of proposed control method.

In order to further verify the validity of the proposed control method, the algorithm of this paper is compared with the simulation results in [12]. So the robustness of trajectory tracking controller against the disturbance and model uncertainties can be evaluated. All the simulation results are depicted in Figures 1–4. Figure 1 shows the trajectory tracking of ship with the given path, and the ship can track and converge to the reference path with more accuracy in [12]. Figure 2 plots the position tracking errors; the along-track and cross-track errors asymptotically converge to zero faster. Figure 3 gives the control inputs response. Surge, sway, yaw velocities, and orientation of ship during the trajectory tracking control process are plotted in Figure 4, which gives a clear insight into the model response involved in nonlinear dynamics.

5. Conclusions

In this paper, we proposed a solution to the course control of underactuated surface vessel. Firstly, the direct adaptive neural network control and its application are introduced. Then the backstepping controller with robust neural network is designed to deal with the uncertain and underactuated characteristics for the ship. Neural networks are adopted to determine the parameters of the unknown part of the ideal virtual control and the ideal control; even the weights of neural network are updated by using adaptive technique. Finally uniform stability for the convergence of tracking errors has been proven through Lyapunov stability theory.

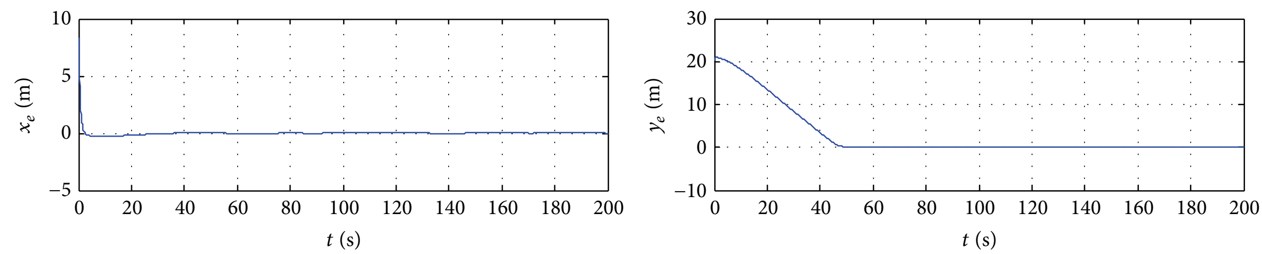

Figure 2: Tracking errors of surge and sway.

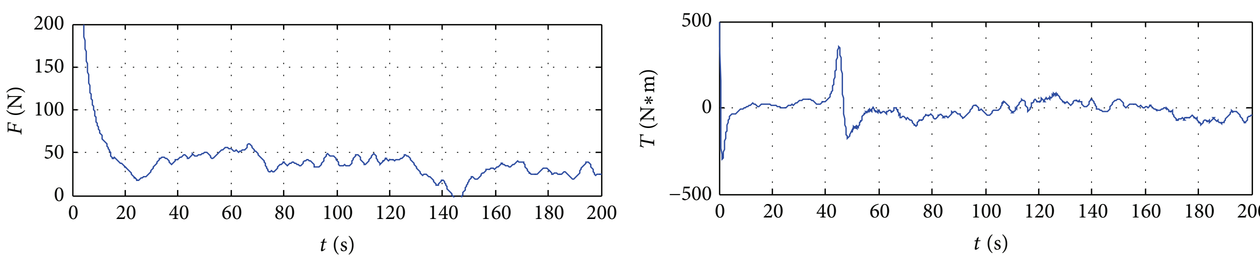

Figure 3: Control force and torque of ship.

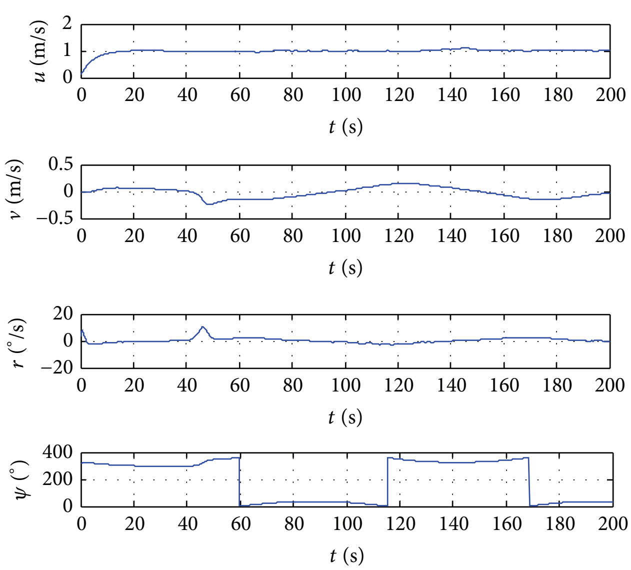

Figure 4: State changing curves of ship.

The simulation results illustrate the performance of the proposed course tracking controller with good precision.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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